Chapter 3

Kinematics Model

3.1 Introduction

In kinematics of manipulators, we study the motion of the links without considering the forces and torques which cause the motion of the links. The robot kinematics can be divided into forward kinematics and inverse kinematics. Forward kinematics is to compute the orientation of the end effector as a function of the joint variables. [9]

Inverse kinematics specifies the end-effector location and computes the associated joint angles or in other word we have orientation position of the end effector so we use it to find angles of three motors.

The relationship between forward and inverse kinematics is illustrated in Figure 3.1.

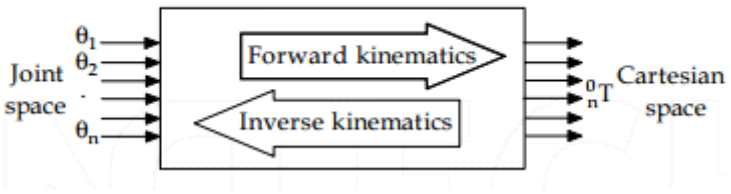

Figure 3.1: relation between inverse and forward kinematics

In general, the forward kinematics (direct) is more complex than inverse kinematics for parallel robot.

3.2 Forward Kinematics

Forward kinematics specifies the joint parameters and computes the configuration of the chain, the aim of direct kinematics is to compute the orientation of the end effector as a function of the joint variables.
The direct kinematics of the *Agile Eye* was solved in [7] and will be reformulated and further analyzed here.
The following constraint equations are written:

$$\sin\psi(\sin\theta_1\sin\phi-\cos\theta\cos\theta_1)+\cos\psi\sin\theta_1\cos\phi=0 \qquad (3.1)$$

$$\cos\psi(\cos\theta_2\sin\theta\cos\phi-\cos\theta\sin\theta_2)+\sin\psi\cos\theta_2\sin\phi=0 \qquad (3.2)$$

$$\sin(\theta_3-\phi)\cos\theta=0 \qquad (3.3)$$

Where θ_i is the motors angle, i=1, 2, 3
And (θ,ϕ,ψ) is the Euler angles.

From equation (3.3) the direct kinematic problem is found to admit two sets of solutions, defined by

$$\cos\theta = 0 \tag{3.4}$$

And

$$\sin(\theta_3 - \phi) = 0 \tag{3.5}$$

A. The first set of solution is trivial solution

Equation (3.4) gives two solution for angle θ:

$$\theta = \pi/2 \quad , \ \theta = -\pi/2$$

From the first solution and after simplification of equation (3.2), the following ,condition is found for arbitrary active joint variables

$$\cos(\phi - \psi) = 0 \tag{3.6}$$

And from the second solution

$$\cos(\phi + \psi) \tag{3.7}$$

These four solution are the trivial solutions to the direct kinematics problem of the Agile Eye robot.

B. The second solution is non-trivial solution

Equation (3.5) gives two solutions for angle ϕ

$$\phi = \theta_3$$

And

$$\phi = \theta_3 \pm \pi$$

Then to find the other angle substitute these two solutions in equation (3.2) and (3.3)

3.3 Inverse Kinematics

To find the set of joint angles that produce a specific end orientation. Inverse kinematics specifies the end-effector orientation and computes the associated joint angles. For parallel manipulators, the specification of the end-effector orientation simplifies the kinematics equations, which yields formulas for the joint parameters.

$$\tan\theta_1 = \frac{\cos\theta\sin\psi}{\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi} \tag{3.8}$$

$$\tan\theta_2 = \frac{\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi}{\cos\theta\cos\psi} \tag{3.9}$$

$$\tan\theta_3 = \tan\phi \tag{3.10}$$